Asian Economic and Financial Review, 2014, 4(9): 1254-1259

Asian Economic and Financial Review

journal homepage: http://www.aessweb.com/journals/5002

CANONICAL COALITION GAME THEORY FOR OPTIMAL PORTFOLIO SELECTION

Habip Kocak

Kent Business School, Universtiy of Kent, Canterbury, UK, Department of Econometrics, Faculty of Economics, Marmara University, TR

ABSTRACT

Special mathematical techniques have been developed in order to analyze conflict-competition situations. Game theory provides a formal analytical framework with a set of mathematical tools to study the complex intersections among rational players (Osborne, 2004). The purpose of developing this theory is to examine the rational ways of behaving for conflicting groups or individuals and to make sure that one of these groups is the winner. Throughout the past decades, game theory has made revolutionary impact on a large number of disciplines ranging from economics, engineering, political science, philosophy or even psychology (Myerson, 1991).Several approaches have been produced to the Portfolio selection problem, which became popular among researchers with the article of Harry M. Markowitz, published in Journal of finance in 1952, which occupies an essential place in the literature. Canonical Coalition Game Theory is among these approaches. In this paper the optimality of a portfolio partnership which will be created by stocks with identical targets but different risk capabilities will be examined with Coalition Game Theory. The obtained optimal gain will be distributed depending on risk coefficients using Shapley vector.

Keywords: Coalition game theory, Portfolio optimisation, Optimisation theory, Stock market, FTSE 100.

JEL Classification: C710, C610, G12.

Contribution/ Originality

This study is one of very few studies which have investigated application of Cooperative Game Theory to Portfolio Selection Problem. This is a new approach to literature, which has applied in London Stock Exchange.

1. INTRODUCTION

The stocks, bonds and other valuable papers belonging to a real or legal person constitute a portfolio. The problem of portfolio selection is based on the question of which investment tools and at what rates will be included in the portfolio. Portfolio selection problem focuses on the problem of which investment tool and at what rate the material resources of an investor will be invested.

A general look at portfolio approaches shows that traditional portfolio approaches were accepted until the post-second world war period (Shenoy and McCarthy, 1998). In traditional approaches, investors believed that they could decrease risk only by increasing the number of stocks in their portfolio without taking into consideration the relation between the returns generated by the stocks in the portfolio. According to this approach which recommends the investors to invest in stocks abundant in type and number which have high expected returns, investors wanted to avoid portfolio risk but the means for measuring and calculating risk were unknown (Reilly and Brown, 1999). In order to overcome this uncertainty, Harry Markowitz published his article titled “portfolio selection” and led the launch of important developments in this area. This marked the beginning of modern portfolio theory (Markowitz, 1952), according to which, merely increasing the number of investment alternatives to include in the portfolio is not sufficient, but the relation and direction of investment alternatives between themselves is also effective. The reason for naming this problem “portfolio selection” in the literature is that the purpose of the investor is not about choosing the investment alternatives which are best individually but about selecting a portfolio which will (i) yield the maximum return with a certain risk or a return with minimum risk and (ii) can act together when brought together.

Sharpe (1971) introduced the “single index model” and attempted to explain the return of stocks with a single factor, namely “market index”. Chen and Ross (1986) introduced “multiple index model”. This model is based on the assumption that the return of stocks is affected by a number of economic factors including interest rates and industrial index in addition to market index (Elton and Gruber, 1995).

Markowitz, as well as Metron Miller and William Sharpe who tried to develop this theory were awarded with Nobel Economics Prize. In recent years different approaches came to the agenda about developing optimal portfolio based on the average variance model of Markowitz. Hence, in 1991 Hiroshi Konno and Hiroaki Yamazaki developed a new model for portfolio optimization (Konno and Yamazaki, 1991).

As envisaged by portfolio management models, an investor who resorts to diversification among alternatives when choosing among securities with the purpose of risk reduction is a risk-reducing strategy; however, it can also result in including low-return securities in the portfolio. The return of a portfolio at the end of investment period cannot be known definitely, as the return of securities which constitute the portfolio at the end of investment period is uncertain. In this case return is a random variable.

Game theory is the generic name given to the analytical methods and decision-development tools of applied mathematics developed for the purpose of examining the observable interaction between individuals or other various structures (i.e.: teams or institutions). It has usage area in a variety of fields of social sciences, especially economics, but also sociology and international relations as well as natural sciences. Game theory examines the results that can occur when individuals, institutions or nations interact with each other in the form of a mathematical game. In game theory usually the players (individuals, teams or participants) are defined with rules determined for their interaction, strategic profiles of players (i.e.: their behaviors or decisions), or the results that they can obtain as a result of these decisions. Game theory is built on the assumption that players are rational. This rationality shows that the players follow the rules of the game and try to win it. By using the game theory, the differences in behaviors of individuals interacting in accordance with defined rules, as well as their coalition, integration and separation behaviours and their reasons.

Game theory is the logical analysis of conflict and cooperation situations and its foundations date back to the 17th century (Straffin, 1993). The first mathematician who interested himself in game theory was Emil Borel; however, the foundations of strategic games were laid by the Minimax theory of John von Neumann in 1928. But the theory did not become popular until the book titled 'The Theory of Games and Economic Behavior' coauthored by Neumann and Oscar Morgenstern in 1944. In this book the applications in economics of game theory were detailed. In general, game theory can be divided into two branches: non-cooperative (Başar and Olsder, 1991) and cooperative game theory (Owen, 1995). Non-cooperative game theory studies the strategic choices resulting from the interactions among competing players, where each player chooses its strategy independently for improving its own performance (utility) or reducing its losses (costs). For solving non-cooperative games, several concepts exist such as the celebrated Nash equilibrium (Saad *et al.*, 2013).

While non-cooperative game theory studies competitive scenarios, cooperative game theory provides analytical tools to study the behavior of rational players when they cooperate. The main branch of cooperative games describes the formation of cooperating groups of players, referred to as coalitions (Myerson, 1991), that can strengthen the players' positions in a game. We restrict our attention to coalitional game theory albeit some other references can include other types of games, such as bargaining, under the umbrella of cooperative games. Coalitional games have also been widely explored in different disciplines such as economics or political science.

2. CANONICAL GAME THEORY

Some members in the $N = \{1,2,3,\ldots,n\}$ subset of players constitute some interest-based groups (partnerships) sometimes for increasing their utilities or perform the tasks that they cannot manage on their own, called Coalition. All members of the S subset denoted as $S \subseteq N$ are coalitions and act as single units. Such coalitions are partnered games which are often seen in our daily lives.

Coalition value (showed as **v**) is the maximum value that the utility (transferable utility) that coalition players can obtain without receiving help from non-coalition players. Partnership or coalition game is shown as $(\boldsymbol{N}, \boldsymbol{v})$ in partnered games (Konno and Yamazaki, 1991).

The most common form of a coalition game is characteristic form (Saad et. al). The value of a game which has a characteristic form with transferable utility is defined as $2^N \longrightarrow \mathbb{R}$. The guaranteed real-value *v* function which assigns the $v(S)$ real number is the characteristic function of the game. The characteristic of these transferable utility (TU) games is evaluated as the benefit that they can adopt an appropriate fairness rule and distribute this obtained value and the represented total utility between coalition members. The amount of utility that a $i \in S$ player can get from $v(S)$ constitutes the return of the player; if it is shown as x_i, it becomes $X = (x_1, x_2, \dots, x_i, \dots, x_n) \in \mathbb{R}^{|S|}$ where $|S|$ represents the cardinality of the series.

Canonic coalition games are the coalition game theory games which find the widest application field. The coalition games whose general features are presented above have to meet two basic characteristics so that it can be canonic: it has to have characteristic form and it has to show superadditivity (Owen, 1995).

Within a coalition, players can any time return to their non-cooperation behaviour so as to obtain the returns of non-cooperation behaviour. The cooperation which will be performed in order to prevent this from happening at least has to guarantee the value that was obtained by the non-integrated coalitions, which is defined as superadditivity (Morris, 1994). Therefore superadditive games are always more profitable. Superadditivity can be defined as follows:

$$v(S_{1,}S_2) \supset \{x \in \mathbb{R}^{|S_1 \cup S_2|} / (x_i)_{i \in S_1} \in v(S_1), (x_j)_{j \in S_2} \in v(S_2)\} \forall S_1 \subset N, \forall S_2 \subset N, \quad \text{s.t. } S_1 \cap S_2 = \phi$$

or

$$v(S_{1,}S_2) \geq v(S_1) + v(S_2) \ \forall S_1 \subset N, \forall S_2 \subset N, \text{ s.t. } S_1 \cap S_2 = \phi$$

In a (N,v) type canonic coalition game, due to superadditivity, the players are inclined to create coalition N. Therefore the core of a canonic game is a series of return allocation which guarantees that no player is inclined to leave N in order to form another S ⊆ N coalition. For a TU game, in the case of N grand coalition, if the $x \in \mathbb{R}^N$ $(N = |N|)$ return vector for dividing $v(N)$ is $\sum_{i \in N} x_i = v(N)$, then the group is rational. This means that the total of utilities obtained from players is equal to the total utility that will be obtained from the game. This means that no player will increase its allocation before it decreases the allocation of other players. If all players can obtain as much return as they used to when they acted individually, and if $x_i v(\{i\}), \forall i$, the return vector X is individual rational. Imputation is mentioned when a return vector meets only the above two conditions.

Let x and y be two imputations for (N,v) game and S ⊆ N a coalition. If $\sum_{i \in S} x_i \leq v(S)$ condition is met for $x_i > y_i, \forall_i \in S$, x covers y through S coalition, which is showed as $x \; dom_S \; y$, where all players of the S coalition prefer x to y and x is realised by the S coalition.

The core of transferable utility games is defined as follows (Nishizaki and Sakawa, 2001):

$$C_{TU} = \{x: \sum_{i\in N} x_i = v(N) \ and \ \sum_{i\in S} x_i \geq v(S), \forall\, S \subseteq N\}$$

In other words, the core is a series of ground where no $S \subseteq N$ coalition has a motive for refusing the offered return allocation, leaving the grand coalition or establishing an S coalition instead. The core guarantees that these deviations will not occur, as any x return allocation that is located in the core guarantees a level of utility which is at least equal to $v(S)$ for each $S \subseteq N$. The core of the game can be an area or a point; it can also be an empty set.

Generally, the existence of a certain transferable utility (N,v) game and $x \in \mathbb{R}^N$ condition transforms into the solution of a linear programming model (Saad *et al.*, 2013).

$$min_x \sum_{i\in S} x_i$$

$$\sum_{i\in S} x_i \geq v(S), \forall S \subseteq N$$

The existence of the core of the game depends on the existence of LP, through which it I sought whether the core is empty or not; NLP leads to the solution as a result of the exponential growth of limitations along with the number of players in N.

The second technique which is offered with the purpose of controlling whether the core is empty is utilizing *Bondareva-Shapley* theorem. The logic of this theorem is buit on the balance of the game. It can be interpreted that the core of a game is not empty if and only if it is in equilibrium (Conitzer and Sandholm, 2008).

In order to overcome such challenges as the emptiness of the core or choosing an appropriate sharing as it is too large or the ability to perform a fair sharing, a solution concept has been sought which can relate each coalition game (N,v) with a unique return vector which is known as “the value of the game”. Shapley defined and characterized some features for this problem. The values titled as Shapley axioms and Shapley vector are defined for transferable functions. They are based on the principle of fair distribution of allocations and each player receives a share proportionate to the contributions they make to the game. The maximum return that coalition S can obtain in the case of $i \in S$ is v(S); the return of the coalition generated by players of an S coalition excluding i is give as $v(S \setminus \{i\})$; hence, the contribution made by the player i to coalition S is $v(S) - v(S \setminus \{i\})$. The probability of existence of S sets which include i is as follows:

$$P(S) = \frac{(n - |S|)!\,(|S| - 1)!}{n!}$$

The utility expected by player i from coalition S is:

$$P(S).\,[v(S) - v(S \setminus \{i\})] = \frac{(n - |S|)!\,(|S| - 1)!}{n!}[v(S) - v(S \setminus \{i\})]$$

The expected utility from all coalitions that obtain return is:

$$\phi_i(v) = \sum_{i\in S} \frac{(n - |S|)!\,(|S| - 1)!}{n!}[v(S) - v(S \setminus \{i\})]$$

The $\phi(v) = (\phi_1(v), \phi_2(v), \ldots, \phi_i(v))$ vector which constitute $\phi_i(v)$ utilities obtained for $\forall_i \in N$ is called the Shapley Vector of the game with a characteristic function is called the Shapley Vector of the game with a characteristic function *v*.

$$\sum_{i=1}^{n} \phi_i(v) = v(N)$$

shows that the total utility that will be obtained from this game will be fairly shared in proportion to their contributions.

3. LITERATURE REVIEW

Coalition game theory has found a place in especially many fields which require coalition. Communication networks (Saad *et al.*, 2013) and particularly wireless networks are coalitional ones and their distribution has to be fair. Saad et al. Explained coalition games with three different methods in their study and suggested a holistic network prepared in accordance with the demands of communication engineers. Mathur et al. Recommend a model for wireless networks in their paper; Cohen et al. Made use of coalition games in the featured choice (Cohen and Vijverberg, 2008).Lemaine dealt with 5 basic applications of game theory for insurance industry (Lemaris, 2013).

Bell and Cover showed the optimization conditions required for optimality portfolio in their 1988 dated article titled “Game Theoretic Optimal Portfolio” (Bell and Cover, 1988).

Implemented the optimal portfolio selection problem to IBOVESPA index which is traded in Brazilian Stock Exchange and obtained highly successful results (Farias *et al.*, 2005).

4. RESEARCH METHODOLOGY

The FTSE-100 Index, also called FTSE-100 (Financial Times and Stock Exchange) is the largest stock exchange in Europe and 4th largest of the world. The FTSE-100 is a share index of the 100 companies listed on the London Stock Exchange with the highest market capitalization. It is one of the most widely used stock indices and is seen as a gauge of business prosperity for business regulated by UK company law. The index is maintained by the FTSE Group, a subsidiary of the London Stock Exchange Group.

In this paper a portfolio with maximum return and minimum risk will be formed from the stock certificates traded in FTSE-100 and the weights of each stock certificate in the portfolio will be calculated. Portfolio selection will be designed as a canonic coalition game and the players will be the stock certificates which will be included in the portfolio as well as the nature player. The return to be obtained at the end of portfolio selection will be distributed in a fair sharing with Shapley Vector. Risk return values of the stock certificates that are traded in FTSE-100 were treated with clustering analysis based on 330 days of operations between January 1st, 2013 and December 1st, 2013 and with the help of SPSS 13.0 package program, and divided into 3 clusters (risk groups)

(Özkok, 2009). It was considered suitable that the best 5 stock certificates should be chosen from each risk group.

The subset of the players who play against Nature (Great Player) is $I = \{A.B.C\}$; each player represent investors with different investing understandings, where A is the investor who avoids risks, B is risk-indifferent investor and C is risk-taking investor. Nature player is to be shown with N who directs the market and is able to change it when it desires.

Table-1. Players and Strategies

Players	Strategy	Code	Name	Sector
	A_1	BATS	BAT	Tobacco
	A_2	CAN	Centrica	Gas, Water & Utl.
PLAYER A	A_3	SSE	SSE	Electricity
	A_4	ULVR	Unilever	Food Producers
	A_5	GSK	GlaxoSmithKline	Pharma & Biotech
	B_1	ARM	ARM	Tech. Hard. Equipment
	B_2	BP	BP	Oil & Gas Producers
PLAYER B	B_3	BARC	Barclays	Banks
	B_4	EZJ	Easy Jet	Travel &Leisure
	B_5	MKS	Marks & Spencer	General Retailers
	C_1	ANTO	Antofagasta	Mining
	C_2	FRES	Frenillo	Mining
PLAYER C	C_3	RBS	Royal Bank of Scotland	Banks
	C_4	TLW	Tullow Oil	Oil & Gas Producers
	C_5	VED	Vedanta Resources	Mining

The Nature Player has three basic strategies, which are determined as follows:

D1: formation of a balanced market,

D2: formation of an unbalanced market

D3: Creating a risky market

Payoff matrixes related to the game played by each player against nature as zero-sum game was structured by consulting to three experts from stock exchange market. The evaluation was determined based on the scale of Saaty (Koçak, 2008). However, as performance evaluations were obtained from three experts, geometric averages were taken so as to minimize the deviation.

$$P_A(A_i, D_r) = a_{ir}, \quad P_B(B_j, D_r) = b_{jr}, \quad P_C(B_k, D_r) = c_{kr}$$

$$i = 1,2, \dots ,5 \quad , j = 1,2, \dots ,5 \ , k = 1,2, \dots ,5 \ , r = 1,2,3$$

Payoff matrixes structured for each player are given as follows:

Table-2. Payoff matrix structured for player A

	D1	D1	D1
A1	4,71769398	5,59344471	6,804092116
A2	6,804092116	6,257324746	5,277632088
A3	6,804092116	6,804092116	3,556893304
A4	6,804092116	8,276772529	4,217163327
A5	6,804092116	3,979057208	7

Table-3. Payoff matrix structured for player B

	D1	D1	D1
B1	4,71769398	5,59344471	6,257324746
B2	5,738793548	5,59344471	6,240251469
B3	5,59344471	6,257324746	5,738793548
B4	5,59344471	7,398636223	6,082201996
B5	6,804092116	3,556893304	7

Table-4. Payoff matrix structured for player C

	D1	D1	D1
C1	4,71769398	5,59344471	6,257324746
C2	7	4,71769398	5,738793548
C3	6,804092116	5,59344471	2,466212074
C4	7,398636223	9	4,217163327
C5	3,27106631	3,556893304	6,257324746

When the games whose payoff matrixes are structured above are solved in WINQSB with the help of linear programming, the optimal values of the individual games of risk-avoiding A, risk-indifferent B and risk-taking C players against player nature are as follows:

$v(\{A\}) = 6{,}01 \quad v(\{B\}) = 6{,}03 \quad v(\{C\}) = 5{,}81$

Payoff matrix belonging to the small coalitions formed by players A and B are given below. Optimal game value was obtained as $v(\{A, B\}) = 12{,}41$ from the solution of linear programming model which was structured accordingly.

Table-5. Payoff matrix of the game structured depending on the small coalition of players A and B

	D1	D2	D3
A1B1	9,435387961	11,18688942	13,06141686
A1B2	10,45648753	11,18688942	13,04434359
A1B3	10,31113869	11,85076946	12,54288566
A1B4	10,31113869	12,99208093	12,88629411
A1B5	11,5217861	9,150338015	13,80409212
A2B1	11,5217861	11,85076946	11,53495683
A2B2	12,54288566	11,85076946	11,51788356
A2B3	12,39753683	12,51464949	11,01642564
A2B4	12,39753683	13,65596097	11,35983408
A2B5	13,60818423	9,81421805	12,27763209
A3B1	11,5217861	12,39753683	9,81421805
A3B2	12,54288566	12,39753683	9,797144774
A3B3	12,39753683	13,06141686	9,295686853
A3B4	12,39753683	14,20272834	9,6390953
A3B5	13,60818423	10,36098542	10,5568933
A4B1	11,5217861	13,87021724	10,47448807
A4B2	12,54288566	13,87021724	10,4574148
A4B3	12,39753683	14,53409727	9,955956875
A4B4	12,39753683	15,67540875	10,29936532
A4B5	13,60818423	11,83366583	11,21716333
A5B1	11,5217861	9,572501918	13,25732475
A5B2	12,54288566	9,572501918	13,24025147
A5B3	12,39753683	10,23638195	12,73879355
A5B4	12,39753683	11,37769343	13,082202
A5B5	13,60818423	7,535950512	14

The payoff matrix belonging to the small coalition 2 formed by players A and C is given below. Optimal game value was obtained as $v(\{A,C\}) = 12{,}08$ from the solution of linear programming model which was structured accordingly.

Table-6. Payoff matrix of the game structured depending on the small coalition of players A and C

	D1	**D2**	**D3**
A1C1	9,435387961	11,18688942	13,06141686
A1C2	11,71769398	10,31113869	12,54288566
A1C3	11,5217861	11,18688942	9,27030419
A1C4	12,1163302	14,59344471	11,02125544
A1C5	7,988760291	9,150338015	13,06141686
A2C1	11,5217861	11,85076946	11,53495683
A2C2	13,80409212	10,97501873	11,01642564
A2C3	13,60818423	11,85076946	7,743844162
A2C4	14,20272834	15,25732475	9,494795414
A2C5	10,07515843	9,81421805	11,53495683
A3C1	11,5217861	12,39753683	9,81421805
A3C2	13,80409212	11,5217861	9,295686853
A3C3	13,60818423	12,39753683	6,023105379
A3C4	14,20272834	15,80409212	7,774056631
A3C5	10,07515843	10,36098542	9,81421805
A4C1	11,5217861	13,87021724	10,47448807
A4C2	13,80409212	12,99446651	9,955956875
A4C3	13,60818423	13,87021724	6,683375401
A4C4	14,20272834	17,27677253	8,434326653
A4C5	10,07515843	11,83366583	10,47448807
A5C1	11,5217861	9,572501918	13,25732475
A5C2	13,80409212	8,696751188	12,73879355
A5C3	13,60818423	9,572501918	9,466212074
A5C4	14,20272834	12,97905721	11,21716333
A5C5	10,07515843	7,535950512	13,25732475

The payoff matrix belonging to the small coalition 3 formed by players B and C is given below. Optimal game value was obtained as $v(\{B,C\}) = 12{,}08$ from the solution of linear programming model which was structured accordingly.

Table-7. Payoff matrix of the game structured depending on the small coalition of players B and C

	D1	**D2**	**D3**
B1C1	9,435387961	11,18688942	12,51464949
B1C2	11,71769398	10,31113869	11,99611829
B1C3	11,5217861	11,18688942	8,72353682
B1C4	12,1163302	14,59344471	10,47448807
B1C5	7,988760291	9,150338015	12,51464949
B2C1	10,45648753	11,18688942	12,49757621
B2C2	12,73879355	10,31113869	11,97904502
B2C3	12,54288566	11,18688942	8,706463543
B2C4	13,13742977	14,59344471	10,4574148
B2C5	9,009859859	9,150338015	12,49757621
B3C1	10,31113869	11,85076946	11,99611829
B3C2	12,59344471	10,97501873	11,4775871
B3C3	12,39753683	11,85076946	8,205005623
B3C4	12,99208093	15,25732475	9,955956875

Continue

B3C5	8,864511021	9,81421805	11,99611829
B4C1	10,31113869	12,99208093	12,33952674
B4C2	12,59344471	12,1163302	11,82099554
B4C3	12,39753683	12,99208093	8,54841407
B4C4	12,99208093	16,39863622	10,29936532
B4C5	8,864511021	10,95552953	12,33952674
B5C1	11,5217861	9,150338015	13,25732475
B5C2	13,80409212	8,274587285	12,73879355
B5C3	13,60818423	9,150338015	9,466212074
B5C4	14,20272834	12,5568933	11,21716333
B5C5	10,07515843	7,113786609	13,25732475

The payoff matrix belonging to the much desired large coalition formed by players A, B and C is given below. Optimal game value was obtained as $v(\{A, B, C\}) = 18{,}63$ from the solution of linear programming model which was structured accordingly.

Table-8. Payoff matrix of the game structured depending on the small coalition of all players

	D1	**D2**	**D3**
A1B1C1	14,15308194	16,78033413	19,31874161
A1B1C2	16,43538796	15,9045834	18,80021041
A1B1C3	16,23948008	16,78033413	15,52762894
A1B1C4	16,83402418	20,18688942	17,27858019
A1B1C5	12,70645427	14,74378273	19,31874161
A1B2C1	15,17418151	16,78033413	19,30166833
A1B2C2	17,45648753	15,9045834	18,78313713
A1B2C3	17,26057964	16,78033413	15,51055566
A1B2C4	17,85512375	20,18688942	17,26150691
A1B2C5	13,72755384	14,74378273	19,30166833
A1B3C1	15,02883267	16,78033413	18,80021041
A1B3C2	17,31113869	15,9045834	18,28167921
A1B3C3	17,11523081	16,78033413	15,00909774
A1B3C4	17,70977491	20,18688942	16,76004899
A1B3C5	13,582205	14,74378273	18,80021041
A1B4C1	15,02883267	16,78033413	19,14361886
A1B4C2	17,31113869	15,9045834	18,62508766
A1B4C3	17,11523081	16,78033413	15,35250619
A1B4C4	17,70977491	20,18688942	17,10345744
A1B4C5	13,582205	14,74378273	19,14361886
A1B5C1	16,23948008	16,78033413	20,06141686
A1B5C2	18,5217861	15,9045834	19,54288566
A1B5C3	18,32587821	16,78033413	16,27030419
A1B5C4	18,92042232	20,18688942	18,02125544
A1B5C5	14,79285241	14,74378273	20,06141686

Continue

A2B1C1	16,23948008	17,44421417	17,79228158
A2B1C2	18,5217861	16,56846344	17,27375038
A2B1C3	18,32587821	17,44421417	14,00116891
A2B1C4	18,92042232	20,85076946	15,75212016
A2B1C5	14,79285241	15,40766276	17,79228158
A2B2C1	17,26057964	17,44421417	17,7752083
A2B2C2	19,54288566	16,56846344	17,25667711
A2B2C3	19,34697778	17,44421417	13,98409563
A2B2C4	19,94152189	20,85076946	15,73504688
A2B2C5	15,81395197	15,40766276	17,7752083
A2B3C1	17,11523081	18,1080942	17,27375038
A2B3C2	19,39753683	17,23234347	16,75521918
A2B3C3	19,20162894	18,1080942	13,48263771
A2B3C4	19,79617305	21,51464949	15,23358896
A2B3C5	15,66860314	16,0715428	17,27375038
A2B4C1	17,11523081	19,24940568	17,61715883
A2B4C2	19,39753683	18,37365495	17,09862763
A2B4C3	19,20162894	19,24940568	13,82604616
A2B4C4	19,79617305	22,65596097	15,57699741
A2B4C5	15,66860314	17,21285427	17,61715883
A2B5C1	18,32587821	15,40766276	18,53495683
A2B5C2	20,60818423	14,53191203	18,01642564
A2B5C3	20,41227635	15,40766276	14,74384416
A2B5C4	21,00682045	18,81421805	16,49479541
A2B5C5	16,87925054	13,37111135	18,53495683
A3B1C1	16,23948008	17,99098154	16,0715428
A3B1C2	18,5217861	17,11523081	15,5530116
A3B1C3	18,32587821	17,99098154	12,28043012
A3B1C4	18,92042232	21,39753683	14,03138138
A3B1C5	14,79285241	15,95443013	16,0715428
A3B2C1	17,26057964	17,99098154	16,05446952
A3B2C2	19,54288566	17,11523081	15,53593832
A3B2C3	19,34697778	17,99098154	12,26335685
A3B2C4	19,94152189	21,39753683	14,0143081
A3B2C5	15,81395197	15,95443013	16,05446952
A3B3C1	17,11523081	18,65486157	15,5530116
A3B3C2	19,39753683	17,77911084	15,0344804
A3B3C3	19,20162894	18,65486157	11,76189893
A3B3C4	19,79617305	22,06141686	13,51285018
A3B3C5	15,66860314	16,61831017	15,5530116

Continue

A3B4C1	17,11523081	19,79617305	15,89642005
A3B4C2	19,39753683	18,92042232	15,37788885
A3B4C3	19,20162894	19,79617305	12,10530737
A3B4C4	19,79617305	23,20272834	13,85625863
A3B4C5	15,66860314	17,75962164	15,89642005
A3B5C1	17,11523081	15,95443013	16,81421805
A3B5C2	19,39753683	15,0786794	16,29568685
A3B5C3	19,20162894	15,95443013	13,02310538
A3B5C4	19,79617305	19,36098542	14,77405663
A3B5C5	15,66860314	13,91787872	16,81421805
A4B1C1	16,23948008	19,46366195	16,73181282
A4B1C2	18,5217861	18,58791122	16,21328162
A4B1C3	18,32587821	19,46366195	12,94070015
A4B1C4	18,92042232	22,87021724	14,6916514
A4B1C5	14,79285241	17,42711054	16,73181282
A4B2C1	17,26057964	19,46366195	16,71473954
A4B2C2	19,54288566	18,58791122	16,19620834
A4B2C3	19,34697778	19,46366195	12,92362687
A4B2C4	19,94152189	22,87021724	14,67457812
A4B2C5	15,81395197	17,42711054	16,71473954
A4B3C1	17,11523081	20,12754199	16,21328162
A4B3C2	19,39753683	19,25179126	15,69475042
A4B3C3	19,20162894	20,12754199	12,42216895
A4B3C4	19,79617305	23,53409727	14,1731202
A4B3C5	15,66860314	18,09099058	16,21328162
A4B4C1	17,11523081	21,26885346	16,55669007
A4B4C2	19,39753683	20,39310273	16,03815887
A4B4C3	19,20162894	21,26885346	12,7655774
A4B4C4	19,79617305	24,67540875	14,51652865
A4B4C5	15,66860314	19,23230206	16,55669007
A4B5C1	18,32587821	17,42711054	17,47448807
A4B5C2	20,60818423	16,55135981	16,95595687
A4B5C3	20,41227635	17,42711054	13,6833754
A4B5C4	21,00682045	20,83366583	15,43432665
A4B5C5	16,87925054	15,39055914	17,47448807
A5B1C1	16,23948008	15,16594663	19,51464949
A5B1C2	18,5217861	14,2901959	18,99611829
A5B1C3	18,32587821	15,16594663	15,72353682
A5B1C4	18,92042232	18,57250192	17,47448807
A5B1C5	14,79285241	13,12939522	19,51464949

Continue

A5B2C1	17,26057964	15,16594663	19,49757621
A5B2C2	19,54288566	14,2901959	18,97904502
A5B2C3	19,34697778	15,16594663	15,70646354
A5B2C4	19,94152189	18,57250192	17,4574148
A5B2C5	15,81395197	13,12939522	19,49757621
A5B3C1	17,11523081	15,82982666	18,99611829
A5B3C2	19,39753683	14,95407593	18,4775871
A5B3C3	19,20162894	15,82982666	15,20500562
A5B3C4	19,79617305	19,23638195	16,95595687
A5B3C5	15,66860314	13,79327526	18,99611829
A5B4C1	17,11523081	16,97113814	19,33952674
A5B4C2	19,39753683	16,09538741	18,82099554
A5B4C3	19,20162894	16,97113814	15,54841407
A5B4C4	19,79617305	20,37769343	17,29936532
A5B4C5	15,66860314	14,93458674	19,33952674
A5B5C1	18,32587821	13,12939522	20,25732475
A5B5C2	20,60818423	12,25364449	19,73879355
A5B5C3	20,41227635	13,12939522	16,46621207
A5B5C4	21,00682045	16,53595051	18,21716333
A5B5C5	16,87925054	11,09284382	20,25732475

The value of each game is at the same time the characteristic functions of the multi-player game designed for a coalition of the players A, B and C. Accordingly all characteristic functions are shown in the table below.

Characteristic Function	v({∅})	v({A})	v({B})	v({C})	v({A, B})	v({A, C})	v({B, C})	v({A, B, C})
Value	0	6,01	6,03	5,81	12,41	12,08	12,08	18,63

The shares belonging to the players with the help of Shapley Vector are realized as follows:

$\emptyset_A(v) = 6{,}295$ $\emptyset_B(v) = 6{,}305$ $\emptyset_C(v) = 6{,}03$

Accordingly the percentages belonging to the players as a result of the normalization of Shapley Vector values are obtained as follows: $P(A) = 0{,}33789587$ $P(B) = 0{,}33843264$ $P(C) = 0{,}3236715$.

When the obtained results are evaluated, it can be seen that the return of the game played by one player and the return of the coalitions displayed 2,85%, 2,75% and 2,2% increases respectively.

In the solution which provides the $v(\{A, B, C\})$ optimal game value, the possibility of occurrence of the strategies which ensure that A_i , B_j and C_k investment tools are chosen by players A, B and C respectively is $x^* y^* z^*$ (*i*=1,2,…,5, *j*=1,2,…,5, *k*=1,2,…5).

The percentage of A_i investment tool in the entire portfolio is:

$T(A_i) = P(A) x_i^* \sum_{j,k} y_j^* z_k^*$ *i*=1,2,…,5

The percentage of B_i investment tool in the entire portfolio is:

$$T(B_j) = P(B) y_j^* \sum_{j,k} x_i^* z_k^* \quad j=1,2,\ldots,5$$

The percentage of C_i investment tool in the entire portfolio is:

$$T(C_k) = P(C_k) z_k^* \sum_{j,k} x_i^* y_j^* \quad k=1,2,\ldots,5$$

When the above formulas are used for relevant calculations, the weights of all the investment tools are calculated. The calculated weights are given in the following table.

Accordingly, the targeted optimal portfolio is obtained. It is recommended that the following percentages of the relevant investment tools are included in this portfolio: 14% of A_3 (SSE), 24% of A_4 (ULVR), and 9% of A_5 (GSK) stock certificates which are among the strategies of risk-avoiding player (player A), 17% of B_2 (BP), 8% of B_4 (EZJ) and 17% of B_5 (MKS) stock certificates which are among the strategies of risk-indifferent player (player B), and 11% of C_4 (TLW) stock certificate which is among the strategies of risk-taking player (player C).

Table-9. Weights of Stock Sertificates in the Portfolio

Players	Strategy	Code	Name	Weight
PLAYER A	A_1	BATS	BAT	0
	A_2	CAN	Centrica	0
	A_3	SSE	SSE	0,14
	A_4	ULVR	Unilever	0,24
	A_5	GSK	GlaxoSmithKline	0,09
PLAYER B	B_1	ARM	ARM	0
	B_2	BP	BP	0,17
	B_3	BARC	Barclays	0
	B_4	EZJ	EasyJet	0,08
	B_5	MKS	Marks&Spencer	0,17
PLAYER C	C_1	ANTO	Antofagasta	0
	C_2	FRES	Frenillo	0
	C_3	RBS	Royal Bank of Scotland	0
	C_4	TLW	Tullow Oil	0,11
	C_5	VED	Vedanta Resources	0

6. CONCLUSION

Man has been in the position of decision-making throughout ages. Sometimes he made use of experiences and sometimes he developed new techniques. Today the decision-making instruments of rational individuals rely upon weighted analytical methods.

Game theory is a method which provides very positive results and perspectives to rational individuals. The efforts to create optimal portfolio revealed the employment of several new techniques. Coalition games attract attention recently with their ability to make preference among investment options belonging to different risk groups.

In this paper a portfolio which can yield the optimal return was built among 15 stock certificates of FTSE 100 with different risk abilities with the help of coalition games and the obtained return was distributed in accordance with the weight of each stock certificate in the portfolio using Shapley Vector.

REFERENCES

Başar, T. and G.J. Olsder, 1991. Dynamic noncooperative game theory. Philadelpiha, USA: SIAM Series in Classics in Applied Mathematics. pp: 1-14.

Bell, R. and M.T. Cover, 1988. Game theoretic optimal portfolios. Management Science, 34(6): 17-40.

Chen, R. and N.R. Ross, 1986. Economic forces and stock market. Journal of Business, 59(3): 383-403.

Cohen, J. and W. Vijverberg, 2008. Applying game theory to library networks. Journal of American Society for Information Science, 31(5): 369-374.

Conitzer, V. and T. Sandholm, 2008. Complexity of determining nonemptiness of the core. CMU,Tech. Rep.CS-02-137.

Elton, E.J. and M.J. Gruber, 1995. Modern portfolio theory and investment analysis. 5th Edn., New York: Wiley. pp: 161-165.

Farias, C.A., W.C. Vieira and M.L. Santos, 2005. Portfolio selection models: Comparative analysis and application to Brazilian stock market. Revista De Econom,a E Agronegocio, 4(3): 387-408.

Koçak, H., 2008. En uygun dershane seçiminde analitik hiyerarşi sürecinin uygulanması. M.Ü.İİBF. Journal, Turkish, 15(2): 868-869.

Konno, H. and H. Yamazaki, 1991. Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market. Management Science, 37(5): 519-531.

Lemaris, J., 2013. Cooperative game theory and its insurance applications. Astin Bulletin, 21(1): 17-40.

Markowitz, H., 1952. Portfolio selection. Journal of Finance, 7(1): 77-91.

Morris, P., 1994. Introduction to game theory. USA: Springer-Verlag. pp: 210-213.

Myerson, R.N., 1991. Game theory, analysis of conflict. Cambridge, USA: Harvard Uni. Press, 5-6: 75.

Nishizaki, H. and K. Sakawa, 2001. Fuzzy and multiobjective games for conflict resolution. New York: Physica-Verlag A Springer-Verlag Company. pp: 22-23.

Osborne, M.J., 2004. An introduction to game theory. New York: Oxford University Press. pp: 13-17.

Owen, G., 1995. Game theory. 3rd Edn., London UK: Academic Press. pp: 103-115.

Özkok, B., 2009. Doğa'ya karşı oynanan oyuncuların ortaklıklarla odemelerini arttırmaları ve portföy seçimi problemlere Bir Uygulama. Unpublished PhD Thesis, İ.Ü. Graduate School of Social Sciences, Management Department, Numeric Methods Branch, Istanbul, 45-49, Turkish.

Reilly, F.K. and K.C. Brown, 1999. Investment analysis and portfolio management. 6th Edn., USA: Harcourt College Publishers. pp: 211-213.

Saad, W., Z. Han, M. Debbah, A. Hjourngnes and T. Başar, 2013. Coalitional game theory for communication networks: A tutorial. Available from www.arxiv.org/pdf/0905.4057.pdf

Sharpe, W.F., 1971. A linear programming approximation for the general portfolio selection problem. Journal of Financial Quantative Analysis 6: 1263-1275.

Shenoy, C. and K.C. McCarthy, 1998. Applied portfolio management. USA: John Wiley & Sons Inc.,1988. pp: 37-40.

Straffin, P.D., 1993. Game theory and strategy. 3rd Printing., Washington, DC: The Mathematical Association of America, 36: 3-15.